Status Seminar 2021

November 29th		November 30th		December 1st	
14:00 – 14:30	Kauers	14:00 – 14:20	Gittenberger	14:00 – 14:20	Krattenthaler
14:30 – 14:50	Schneider	14:20 – 14:40	Buchacher	14:20 – 14:50	Albion
14:50 – 15:10	Wei	14:40 – 15:00	Hainzl	14:50 – 15:20	Aigner
Break		Break		Break	
15:20 – 15:50	Paule	15:10 – 15:30	Koutschan	15:30 – 16:00	Smoot
15:50 – 16:10	Nuspl	15:30 – 15:45	Wong	16:00 – 16:20	Banerjee
16:10 – 16:30	Drmota	15:45 – 16:05	Schlosser		
		16:05 – 16:35	*Open Problems:* Hainzl Höngesberg Smoot		

Florian Aigner – *Alternating sign matrices and totally symmetric plane partitions*
I will present a new family of symmetric functions related to alternating sign matrices, totally symmetric plane partitions and d-descending plane partitions. We will explore this family by using a Mathematica package for symmetric functions. This is joint work with I. Fischer.

Seamus Albion – *Elliptic A_n Selberg integrals.*
I will present a new elliptic extension of the Selberg integral associated to the Lie algebra A_n. The derivation of this formula is based on the elliptic interpolation kernel, a powerful new tool in the theory of elliptic special functions, which I will also describe.

Koustav Banerjee – *Inequalities for the modified Bessel function of first kind*
In this talk, we will describe how from the study of asymptotics of modified Bessel function of first kind of non-negative order, one can comes up with a family of inequalities that finally leads to answer combinatorial properties, for example log-concavity, higher order Turán inequality of certain arithmetic sequences arising from Dedekind eta quotients.

Manfred Buchacher – *The orbit-sum method – generalized and automatized.*
The orbit-sum method was introduced by Bousquet-Mélou and Mishna to solve certain linear discrete differential equations that arise in the enumeration of restricted lattice walks. Bostan, Bousquet-Mélou, and Melczer indicated how it generalizes to solve linear discrete differential equations not necessarily of order 1. We generalize the orbit-sum method further and explain how it can be applied algorithmically. This is joint work with Manuel Kauers.

Michael Drmota – *Cores and maximal components of planar cubic maps.*
It is already a classical result that the largest components (for example connected components or 2-connected components) in random planar maps have linear expected size and the limiting distribution follows an Airy law. This was systematically studied by Banderier, Flajolet, Schaeffer, and Soria (2001) in the context of critical singularity schemes of generating functions. The main purpose

of the present work is to extend these techniques to more general situations, in particular to cores of random cubic maps, where several additional technical difficulties appear, as well in the combinatorial as well in the analytic part. This is joint work with Marc Noy, Clement Requile, Juanjo Rue.

Bernhard Gittenberger – *TBA.*

Eva-Maria Hainzl – *Universal Asymptotics for Positive Catalytic Equations.*
Functional equations with catalytic variables appear in several combinatorial applications, for instance in the enumeration of lattice paths and in the enumeration of planar maps. In this talk, we will discuss universal asymptotic behaviour of solutions to positive polynomial equations with first and second order discrete derivatives. Similar to previous studied cases, we distinguish between solutions to linear equations (where the subexponential growth of the coefficients is $n^{-3/2}$) and solutions to non-linear catalytic equations (where we have $n^{-5/2}$).

Manuel Kauers – *CAD*

Christoph Koutschan – *Apéry limits and irrationality proofs*
Motivated by Apéry's pioneering proof of the irrationality of $\zeta(3)$, and by Beukers' reinterpretation in terms of integrals, we introduce the notion of Apéry limit. We perform a systematic search of Beukers integrals in order to construct more miraculous irrationality proofs à la Apéry. Moreover, we give a construction that realises certain quotients of L-values of elliptic curves as Apéry limits. This is joint work with Robert Dougherty-Bliss, Doron Zeilberger, and Wadim Zudilin.

Christian Krattenthaler – *The Selberg integral and (some of) its generalisations - introduction to Seamus Albion's talk.*

Philipp Nuspl – *C^2-finite Sequences: a Generalization of D-finite Sequences*
We define sequences which satisfy a linear recurrence with coefficients that satisfy a linear recurrence with constant coefficients themselves. These C^2-finite sequences are a natural generalization of D-finite sequences. We study their computational properties and investigate the similarities and differences compared to the well-known D-finite sequences. A large part of the theory carries over to the C^2-finite case. However, it turns out that the computational aspects are much more involved and are related to difficult problems in number theory.

Peter Paule – *Modular Forms and Holonomic Differential Equations.*
The talk introduces the Algorithm ModFormDE for proving differential equations for modular forms, a joint development in collaboration with Silviu Radu. One possible usage of this algorithm is in the framework of a "first guess, then prove" strategy. Applications, for instance, concern identities in connection with approximations to pi (work of Jon and Peter Borwein), Fricke-Klein relations, or irrationality proofs a la Beukers.

Michael Schlosser – *Some conjectured precise sign patterns for the q-series coefficients in certain finite and infinite products.*
The famous three Borwein Conjectures are statements about precise sign patterns for the q-series coefficients of certain sequences of finite products. While they are simple to state they have long resisted any proof, until recently Chen Wang (University of Vienna) settled the First Borwein Conjecture using asymptotic machinery. In my talk I will review the three Borwein conjectures together with a closely related conjecture by Chen Wang, and present a couple of similar conjectures.

Carsten Schneider – *Solving Linear Difference Equations in Difference Rings.*

Nicolas Allen Smoot – *The Localization Method Applied to Partition Congruences.*
We provide a brief survey of how the localization method can be used to derive interesting parti-

tion identities and congruence properties. In particular, we examine three examples of congruence families. The method has a well-understood theoretical foundation, and can often be used to prove congruences that are otherwise not easy to prove.

Yarong Wei – *Solutions to the First Order Difference Equations in the Bivariate Difference Field.*
Inspired by Karr's algorithm, we consider the summations involving a sequence satisfying a recurrence of order two. The structure of such summations provides an algebraic framework for solving the difference equations of form $a\sigma(g) + bg = f$ in the bivariate difference field $(\mathbb{F}(\alpha, \beta), \sigma)$, where $a, b, f \in \mathbb{F}(\alpha, \beta) \setminus \{0\}$ are known binary functions of α, β, and α, β are two algebraically independent transcendental elements, σ is a transformation that satisfies $\sigma(\alpha) = \beta$, $\sigma(\beta) = u\alpha + v\beta$, where $u, v \neq 0 \in \mathbb{F}$. Based on it, we then describe algorithms for finding the universal denominator for those equations in the bivariate difference field.

Elaine Wong – *Multivariate Asymptotics in Sagemath.*
In this talk, we demonstrate some examples of an automated computation of dominant asymptotics for the diagonal coefficients of multivariate rational functions (in the combinatorial case) using a recent SageMath implementation of algorithms from a 2021 paper of Melczer and Salvy. This is joint work with Benjamin Hackl and Jesse Selover in the context of an AMS Mathematics Research Community.

Open problems

- Eva-Maria Hainzl – *Battleships and minimal transversals in hypergraphs*
- Hans Höngesberg – *A conjectural constant term identity related to vertically symmetric alternating sign triangles.*
- Nicolas Allen Smoot – *Partition Congruences and Gröbner Bases.*